MID-CAREER TEACHER LEARNING IN A MATHEMATICS PROFESSIONAL DEVELOPMENT PROGRAM: FOCUSING ON THE GROWTH OF TEACHERS' PERSPECTIVES

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Abstract

It is important to understand what and how mathematics teachers learn in mathematics professional development (MPD) programs. In this paper, we examined mid-career teachers' learning in our program wherein participant teachers conducted lesson study and analyzed students' learning by collecting and discussing multiple sources of data. We approached the issue of teacher learning from the aspect of broadening teachers' perspectives on teaching and learning in a mathematics lesson. By examining the case of two teachers and focusing on their practical research activity in a lesson on transformations of geometrical figures, we identified four perspectives. These perspectives emerged and developed in different activities and at different periods between the planning of the lesson and the presentation of the final report. The process of growth was further examined using the construct of contextualization. An analysis of two sessions held immediately after the research lesson revealed that the teachers used the four perspectives to recontextualize or decontextualize several classroom situations and that dialogic interaction made it possible for the teachers to form novel interpretations of the behaviors and utterances of students and to reflect on their own instructional actions. This paper thus provides information on features of mid-career teacher learning in a university MPD program that emphasized practical research activity.

Key words: mid-career teacher, teacher learning, perspective, contextualizing, lesson study

INTRODUCTION

This paper addresses the issue of what and how practicing teachers learn in a mathematics professional development (MPD) program. When considering teachers' learning from a career-long perspective, we observe certain tasks and needs that are characteristics of each period (Brown & Borko, 1992). Our focus is on the learning of two mid-career teachers who had been teaching mathematics since 15–20 years.

Features of effective in-service MPD have been synthesized by researchers (Borko, 2004; Sowder, 2007; Sztajn, Borko, & Smith, 2017; Sztajn, Wilson, Edgington, & Myers, 2014; Tirosh & Graeber, 2003; Zaslavsky, Chapman, & Leikin, 2003). These features include focusing on content and students' learning, providing teachers with active opportunities for learning, ensuring coherent and sustained support, and fostering collective participation (Sztajn et al., 2017). However, beyond these broad features, the details of teacher learning from MPD have not been studied in depth and remain unclear. Goldsmith, Doerr, and Lewis (2014) state, "A more intentional and systematic focus on illuminating the black box of teachers' learning might allow us to develop more general understandings about how certain catalysts for change affect the pathways of teachers' learning over time" (p. 21).

This paper enhances our understanding of the nature of teacher learning based on the analysis of two junior-high school teachers who participated in an MPD at a university in Japan. In particular, we are interested in the impact of teachers' practical research activity on the development of their knowledge, teaching practice, and so on. According to Miyakawa and Winsløw (2019), the term *practical research (jissen kenkyu)* is "a broader term that denotes the study and research on teaching practices, carried out mainly by an individual teacher or a group of teachers for the purpose of improving their teaching practices" (p. 283). Despite the acknowledgement of interplay between theory and practice to foster the development of professional knowledge (Huang & Shimizu, 2016; Ruthven, 2002; Ruthven & Goodchild, 2003; Sztajn et al., 2017), the reality of the interplay and its contribution to the development of professional knowledge have not been adequately captured. We attempt to gain an understanding of the details of this development in an MPD program that emphasized practical research activity.

INTERPLAY BETWEEN THEORY AND TEACHING PRACTICE FOR PROFESSIONAL DEVELOPMENT

The dynamic between theory and teaching practice has been pointed out as being a facilitator in the development of professional knowledge in mathematics teaching (Huang & Shimizu, 2016; Potari, 2013; Ruthven & Goodchild, 2003). In an MPD program, such a dynamic is often taken into account; however, realization of the interplay between theory and teaching practice is not uniform across MPD programs. It is often the case that selected theories are presented by the researcher in the program, and the participant teachers then apply the theory when they teach mathematics. Recently, this has included rich reflective activities (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Sztajn et al., 2014; Tsamir, 2008). There are also cases in which a collaborative approach between teachers and researchers is adopted for the analysis of teaching practices to co-construct outcomes and build theory (Hennessy & Deaney, 2009).

In the lesson study (LS) with which this paper is concerned, interplay between theory and teaching practice also varies. The LS is a practice-based, research-oriented, and collaborative style of professional development (Fernandez & Yoshida, 2004; Huang & Shimizu, 2016; Quaresma, Winsløw, Clivaz, da Ponte, Shuilleabhain, & Takahashi, 2018; Stigler & Hiebert, 1999). An interplay with a primary direction from theory to teaching practice is found in *Learning Studies* conducted in Hong Kong and Sweden in which researchers used a specific theory, namely, variation theory (Marton & Booth, 1997), focusing on intended

learning goals. In those studies, the theory guides teaching practice, and plays a fundamental role throughout the LS process. On the other hand, the Japanese LS does not always explicitly mention a specific theory. However, when teachers take a cyclical approach to teaching (Fernandez & Yoshida, 2004; Lewis & Hurd, 2011), questions and hypotheses about their own teaching or, more broadly, about issues of teaching and learning of mathematics, are formulated by teachers. Fujii (2015) describes the critical dimensions of task and task design in a LS, which can be said to include teachers developing conjectures regarding students' construction of mathematical knowledge by referencing findings and models in previous studies (see also Takahashi, 2006). In the LS cycle, the interplay between theory and teaching practice might be taking place, but it is not clearly captured.

In addition to these variations in the interplay between theory and teaching practice, another aspect which is not clear is the process of teachers' development of professional knowledge when they engage in such interplay in their MPD program, including the LS. Furthermore, important questions may relate to voluntariness: Is the interplay voluntarily created by teachers, and what are the conditions that enable teachers to voluntarily foster the interplay? Although not directly answering these questions, there are some studies that point to teachers' own research activities as a driving force for the interplay and for those teachers' development of professional knowledge.

Iwasaki and Miyakawa (2015) examined a two-year graduate program comparative to a master's program in Japan. This newly developed MPD program, called the Professional Degree Program, emphasizes field practice that includes teaching, observing, and analyzing lessons. Through the examination of changes in a teacher's discourse in their logs on observed mathematics teaching practices over the two years, the researchers classified the discourse into four categories: empirical, practical, quasi-theoretical, and theoretical. Iwasaki and Miyakawa (2015) further highlight the essential role of university researchers and the activity of writing a research paper. They conclude that "in order to place theories at teachers' disposal, it would not be enough to provide course work to learn them and long-term practicum" (p. 96). Miyakawa and Winsløw (2019) also question seemingly simplified operationalizations of a LS where such a study is handled as an isolated cyclic activity. They argue that a LS is, in fact, only an element of a comprehensive Japanese *paradidactic infrastructure* for professional exchange and publication of teacher knowledge. Basing their argument on the Anthropological Theory of the Didactic, they articulate how a teacher's practical research activity is stimulated and maintained by infrastructural conditions other than the live observation of and reflection on lessons. Their call for broadening this vision, especially their attention to the opportunity for presentation and publication of teachers' knowledge, is informative when we look at teachers' learning. It suggests that teachers' voluntary action regarding the interplay should be exhibited more explicitly in a broader context that includes both teaching and research. The importance of a wider context in the transformation of teaching activity is also explored by Potari (2013) using the framework of *Activity Theory*. By distinguishing two activity systems, research and teaching, Potari illustrates the development of links made by a group of teachers. Interconnections and contradictions between the systems are pointed out as important sources of change and development for teachers.

FRAMEWORK OF THE STUDY

The authors of this paper have designed an MPD that encompasses teachers' practical research activity by applying the triangle framework constructed by Silver and Herbst (2007) (Hino & Makino, 2012). Silver and Herbst constructed the scholarship triangle in the field of mathematics education by stressing the connections between research and practice, each of which relates to a set of problems. In their triangle, theory is placed at the center and plays a mediating role in connecting any two of the three components (the three vertices of the triangle). We localized the triangle by placing the teachers' mathematics lesson framework (MLF) at the center and three specific components (the teacher's problems or concerns, the variety of research in mathematics education, and the teacher's teaching practice) at the three vertices (see Figure 1). MLF is a conceptual and interpretive model of teaching and learning in a mathematics lesson; it is constructed by the group of participating teachers. It includes mathematics teaching objectives, lesson organization principles and the teacher's role during the lesson. It has a connection to research and practice, and is also influenced by teachers' problems or concerns (see the two types of lines in Figure 1). In previous papers, we showed examples of the MLFs created by several groups of participant teachers; we also showed the results of a survey administered to the teachers who had completed our program and noted some merits of collaborative creation of an MLF among teachers (Hino & Makino, 2012, 2014).

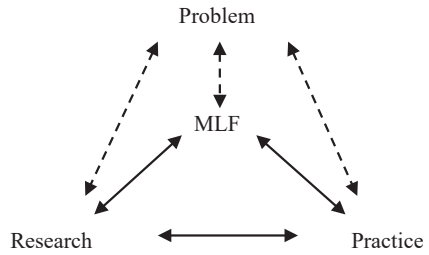

Figure 1. MLF and the practice-based inquiry triangle

In this paper, we continue to use our inquiry triangle framework to accomplish our aim. The focus this time is on the process of teachers' creation of an MLF. We approach the process by building further on two ideas: one is perspective and the other is contextualizing. Both ideas come mainly from the work done by Lin and his colleagues (Lin, Yang, Hsu, & Chen, 2018).

Lin et al. (2018) explore perspectives on the use of theory by mathematics teacher educators and researchers (MTE-Rs) to facilitate the development of teachers. Using the triangle proposed by Silver and Herbst (2007), a qualitative analysis of discussions and presentations in a 2-day educator forum was carried out, and Lin et al. (2018) identified three distinct perspectives: a perspective focusing on research, a perspective focusing on practice, and a perspective on the connection between research and practice. Furthermore, they identified context as an important component, and divided the last perspective into two sub-categories: connection with consideration of context, and no connection with consideration of context. It is noteworthy that the former sub-category considers contexts specific to teachers' teaching practices as being relevant to their learning of theory. Perspectives in this category emphasize the importance of considering the cultural characteristics of teaching and learning, communities that influence the facilitation

of teachers' learning of theory, and teachers as the center of learning and as the inventors of theory rather than mere consumers thereof. Lin et al. (2018) further show that a core issue of this perspective is MTE-Rs' activities of contextualizing. Drawing on the study by van Oers (1998), they argue that the activity of contextualizing by MTE-Rs helps facilitate teachers toward learning theory.

While Lin et al. (2018) employed perspective regarding MTE-Rs' use of theory, we considered perspective as a lens to examine the growth of the teachers' ideas that constitute the MLF while they engage in their practical research activity. Lin et al. (2018) define perspective as a frame of reference to "specifically describe how one deals with a situation by coordinating a set of ideas and actions" (p. 198). Citing Romberg (1992), they stress that perspective is not just a self-reported statement of ideology or attitudes but encompasses specific behaviors based on one's beliefs or assumptions. A reason for using the construct of perspective is that it contributes to exploring core ideas that anchor teachers in viewing and dealing with specific situations in their practices. Some perspective is closely related to a teacher's previous teaching experiences. It may also be the case that perspective strongly reflects a teacher's knowledge of certain research. Both perceptions are applicable in as far as they serve the teacher in determining how to deal with and behave in a particular situation. We describe the growth of teachers' ideas of MLF through the emergence and development of several perspectives.

We use the construct *contextualizing* to look further into the process of creation of an MLF. van Oers (1998) views *context* as cultural activity and states, "What counts as a context depends on how a situation is interpreted in terms of activity to be carried out" (p. 481). Context is constructed for an individual by "determining his particular goal, examining his prior experiences, finding out which means are available, investigating which actions make sense to perform in order to achieve the goal chosen, and by relating motive, goal, object, means etc." (pp. 481-482). van Oers continues that "the basic process here is the process of *context making* (which I will call *contextualizing*), which is an intellectual activity by itself, embedded in a current sociocultural activity" (p. 482, emphasis in the original). As subconstructs of contextualizing, Lin et al. (2018) state that "decontextualizing refers to the process of deriving core ideas from a certain situation, whereas recontextualizing refers to the coordination between the core ideas and the new encountered situation" (p. 209). We believe that the process of *contextualizing* will offer a useful tool to describe the mechanism of the development of perspectives. This is because when teachers create context from a situation, the teachers' perspective has the possibility to play a role; it is also possible that the perspective itself is affected by such context-making acts. Such details may illuminate the creation of an MLF in relation to the interplay between perspectives and various situations in teaching practice.

Therefore, this paper pursues the following two research questions:

- What perspectives on teaching/learning in mathematics lesson emerge and develop among teachers while they engage in practical research activity in a university MPD program?
- How does a teacher's process of contextualizing situations function in the emergence and development of these perspectives?

THE *NAICHI-RYUGAKU* PROGRAM: A UNIVERSITY-PREFECTURAL BOARD OF EDUCATION COLLABORATIVE PROFESSIONAL DEVELOPMENT PROJECT IN JAPAN

In Japan, teachers are required by law to conduct research and self-improvement, and are ensured an opportunity for in-service education. There are many professional development (PD) programs at national, prefectural and municipal levels (MEXT, 2020). Through these programs and through everyday practices, teachers acquire and maintain skills, knowledge and pedagogical values. Many PD programs are conducted at professional institutions under the leadership of local boards of education. With respect to university-associated PD programs for practicing teachers, there are four major types: (1) a set of lectures that are requisite for the periodic renewal of teaching licenses (every 10 years), (2) traditional master's programs, (3) newly developed professional degree programs (mentioned earlier in the study by Iwasaki & Miyakawa, 2015), and (4) university PD in cooperation with prefectural boards of education. Programs (2)–(4) are not mandatory but elective. Teachers are typically nominated by the board of education as candidates for participation in these programs.

About the program

Our MPD program belongs to type (4) described above. It is part of a larger in-service education project called the *Naichi-Ryugaku* program (meaning "to go to a foreign university to learn") involving cooperation between educational institutions, including our university, and the Tochigi prefectural board of education. Although this type of in-service education has a long history in Japan, ways of implementing the program vary according to local government preferences.

In the Tochigi prefecture, when teachers are selected as participants by the board of education, they are given leave from their jobs for six months (or, in the case of high school teachers, for one year). Participants can choose their major fields of study and engage in studies in those fields. After the completion of the program, they often play a role as middle leaders in their schools. Some are promoted to the position of teacher educators and work in local boards of education in the prefecture.

In our mathematics department, we usually work with two or three teachers in each six-month term. They commonly have 15–20 years of teaching experience in either elementary or junior-high schools. As the teachers come from different areas and different school levels, this program gives them a unique opportunity to study mathematics and mathematics teaching collaboratively with members of a larger community.

Major activities and practical research

In our MPD program, we offer the participating teachers three major opportunities to improve their professional knowledge and skills. First, they can freely attend both undergraduate and graduate mathematics education courses and seminars. Second, they can join us when we are called on by local elementary or junior-high schools to be advisors in their lesson studies; they usually observe a research lesson and participate in the debriefing with us and other teachers who are involved in the LS. Third, the teachers conduct their own practical research activities, and it is this opportunity that is the focus of this paper. The teachers' practical research includes the LS cycle of planning, conducting and reflecting on lessons. It also includes collecting and analyzing lesson data, and writing a final report to be submitted to the board of

education. In addition, after completing our program, these teachers usually present their results at local teacher conferences or meetings.

For the purpose of our study, we organized participants' activities in practical research around three principles (see also Hino & Makino, 2012, 2014). First, we began the session by asking participants to share their experiences, concerns, or problems and encouraging them to formulate a research theme incorporating their areas of interest. Fujii (2015) stresses the importance of educational value underpinning phases of a LS cycle. We believe that eliciting teachers' areas of interest drives them to recognize educational values more deeply. Second, teachers were asked to create MLFs that were compatible with their specific research themes by designing, conducting, and reflecting on a research lesson. In the early stages, we asked them to present their ideas as to what research lessons they wished to do. After they had conducted the research lessons, we asked them to examine the results of those lessons. Even though the content and path of creation of MLFs vary according to the groups of participant teachers, MLF is often included in teachers' final reports. Third, the researchers maintained a dialogue with participant teachers throughout the sessions. The three university researchers who are the authors of this paper varied with respect to length of career in research, LS experience, and area of research interest. All those concerned with this study express and exchange their opinions in the session discussions (Hino & Makino, 2015).

A CASE STUDY OF TWO JUNIOR-HIGH SCHOOL TEACHERS

Profiles of the two teachers

Two mathematics teachers (Mr. T and Mr. M) who worked in different junior-high schools participated in our MPD from October 2018 to March 2019. Both of them had accumulated teaching experience of approximately 15 years in several junior-high schools, although they had not met each other before.

The teachers had specific motivation for participating in our program. Mr. T's goal in mathematics teaching was to nurture student's identity by learning the importance of cooperation with other people in the lessons. He also wanted students to express their thinking and reasoning as well as to enjoy learning mathematics. Mr. T was interested in teaching mathematics by fostering students' interaction and mutual learning. He said that through our program, he wanted to learn a sustainable method of teaching mathematics that he could implement in daily lessons by modifying his developed method of teaching.

Mr. M became a mathematics teacher after working as a system engineer and had 12 years of teaching experience. The schools he taught in were always small in size, and he would be the only mathematics teacher. Because of these circumstances, he was concerned whether his way of teaching was effective for his students. He said that he wished to learn what was required for mathematics teaching nowadays. In particular, by reading literature on the new Japanese Course of Study (MEXT, 2018), he had come to be interested in ways to incorporate the process of finding and solving mathematical problems into his lessons to foster students' autonomy in learning.

Activities for practical research in the 2018 second-semester program

With regard to practical research, we conducted 20 sessions. These included meetings between the three researchers and the two teachers, small meetings with some of the members of the research group, and on-site meetings at Mr. T's school that included the conducting of the research lesson. In the first session involving all five members of the group, we discovered that Mr. T and Mr. M had similar concerns about fostering students' abilities to think, make decision, and express their thinking through mathematics lessons, which became the research theme the teachers chose to start with.

It was also found that both the teachers were interested in the subject of geometry, and they quickly began to develop their vision of the research lesson on the chapters on geometry. First, each teacher developed his own lesson plan. However, since it turned out that Mr. M could not conduct his research lesson due to a development at his school, he assisted Mr. T in preparing the lesson plan. As a consequence, the two teachers collaboratively developed, conducted, and analyzed a research lesson in Mr. T's seventh grade class. Table 1 summarizes the duration and content of the major activities that the teachers engaged in as part of their practical research.

Table 1. Period and content of major activities in practical research

Months	Major activity	Content of activity
October	Discussing an initial idea for a lesson	Teachers explained their experience, problems, concerns, and interest in mathematics teaching. They also described their aims in teaching mathematics as well as how they wanted to approach the lesson that they would conduct during the program.
October-December	Designing a research lesson	The teachers collaboratively designed a research lesson in a seventh grade classroom on the textbook chapter covering transformations of geometrical figures. The students were expected to apply their knowledge of transformations of geometrical figures (parallel translation, symmetric transformation, and rotational transformation) to achieve their goal in the activity.
December	Conducting the research lesson	The research lesson, which comprised two consecutive lessons, was conducted on December 20 by Mr. T. The lesson was recorded on video. Students' behavior in the group work was also recorded via video and audio. Mr. T, Mr. M, and a university researcher (the third author of this paper) observed the lesson.
December	Collecting data by interviewing students	Immediately after the lesson, the two teachers and the researcher reflected on the lesson and chose several students to be interviewed. The teachers conducted the interviews over a period of two days. They then came together and reflected on what the students had said in the interviews.
January-February	Examining the effect of the lesson by analyzing data	Over the course of several sessions, the five group members analyzed the data to examine the students' learning based on the students' group activities. Different members expressed their opinions and proposed several ideas for understanding the process and product of students' learning in the lesson.
February-March	Summarizing the content of the inquiry in the form of a report	The teachers reflected on all the activities and summarized the content of their inquiries in the form of a report. The theme of the practical research was finalized as "Fostering students' critical thinking through evaluation and revision of explanation in junior-high school mathematics."

Flow of Research lesson

The research lesson in a seventh grade class (29 students) centered on the application of transformations of geometrical figures. The goal of the lesson was as follows: "Using transformations of geometrical figures to create a better instruction manual to help classmates recreate an original figure." The overview of the two consecutive lessons is as follows:

Lesson 1 (50 minutes)

1. The teacher introduced traditional and well-known fretwork in the Kanuma area where the school was located. He then motivated students in relation to the need for an instruction manual to recreate the work.
2. Students engaged in a recreation game. Working in pairs, one student was shown a figure and asked to explain its construction to the other student. The other student then attempted to recreate the figure. Students were expected to experience difficulty in recreating the original figure and thus feel the need for clarifications.

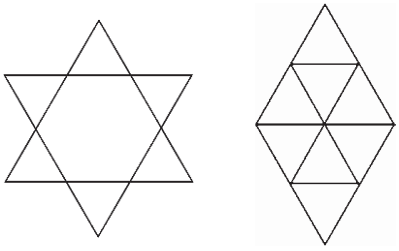

Figure 2. Figures for instruction

3. The teacher stated the task for the day, namely, to create an instruction manual that explained how to draw a figure by using transformations of geometrical figures.
4. Students were told about the goal of the lesson and the points they should pay attention to when using transformations of geometrical figures so that their classmates could recreate the figure.
5. Six groups of students (four students in each group) were formed. Three groups were assigned the left-hand figure, as shown in Figure 2, and the rest of the group was assigned the right-hand figure. Each group worked together to create an instruction manual. The parts of the figures were also provided in concrete form in case students needed to manipulate them to get some ideas.
6. Students reflected on Lesson 1.

Lesson 2 (50 minutes)

1. Students exchanged their instruction manuals between groups. They recreated the figures alternately by reading another group's instruction manual. They were then shown the original figures to check for correctness.
2. Each group thoroughly studied the instruction manual they had just used. They made comments on it and gave it back to the group who had created it.
3. Using the comments, each group evaluated their instruction manual and modified it.
4. Students noted important points to create better instruction manuals that would help others recreate the figures.
5. As a whole-class activity, students summarized what a better instruction manual should include.
6. Students reflected on Lesson 2.

Data collection and analysis

In order to understand the process of learning on the part of the two teachers in the six months, we made use of the following data: a final report written by the two teachers (Takita & Mizuno, 2019); resumes prepared by the teachers for discussion in the sessions; personal notes developed by Mr. M when the two teachers discussed their ideas and questions informally; video recordings and transcriptions of the research lesson (session 9); and transcriptions of audio recordings of sessions 10, 12, and 20. Apart from these sessions, we did not video-record or audio-record our discussion.

In the analysis, we first summarized the entire activity of the practical research, namely 20 sessions, to determine the flow of our discussion. Transcribed data were also segmented to determine the object of discussion. The collected data were then analyzed qualitatively to identify teachers' perspectives during the discussions. Our strategy was to extract core ideas on teaching and learning in the lesson from the written data on our discussion. We started with an analysis of the resumes, and in particular, the goal statement and the description of the development of the research lesson that Mr. T had proposed in sessions 1 to 8. We searched for key words and phrases and determined when they first appeared and whether they disappeared or were maintained. Then we extended the analysis to other parts of the resume and the transcribed records of sessions 10 and 12. After several revisions, we finally synthesized the teachers' perspectives as four core ideas that anchored and guided the teachers when they dealt with issues that they encountered during the practical research activity.

From the transcribed data of sessions 10 and 12, we then analyzed the teachers' ways of using these perspectives. In these sessions, the teachers and a researcher discussed which students should be interviewed (session 10) and what could be learned from these students (session 12). Their roles in the discussion were coded and compared between the teachers and the researcher. Following this, we interpreted the teachers' process of gaining new ideas using the framework of contextualizing (Lin et al., 2018; van Oers, 1998).

All these analyses were done by the three authors collaboratively. The first author conducted the first analysis which included the coding of discussions, after which the second and third authors checked the assigned codes and the analysis. Where there were discrepancies, the three authors discussed them and made further modifications. In addition, the third author contributed comments on the codes and analysis of sessions 10 and 12 by reflecting on his thinking in these sessions.

RESULTS

The teachers' perspectives on teaching and learning in a mathematics lesson

From the analysis of the teachers' viewpoints, we identified four perspectives on an MLF that foster students' abilities to think, make decision or express their thinking. They are "using the language of mathematics" (we shorten it as "language"), "revising the initial method to create a better one" ("revision"), "thinking with doubt" ("doubt"), and "change in students" ("change"). Table 2 summarizes descriptions of these perspectives.

Table 2. The teachers' perspectives on teaching and learning in the lesson as identified in their practical research activity

Using the language of mathematics (language): Mathematics provides a common and universal language that makes it possible to convey one's intentions clearly to others. The teachers aimed to enhance students' use of transformations of geometrical figures for the purpose of making communication clear, specific or accurate. They also considered whether students used the language of mathematics learned in the lesson, reasons for their use of such language, and the importance of using this language.
Revising the initial method to create a better one (revision): Revising one's initial method to create a better one is an activity that provides students with the opportunity to reflect on their thinking. This constituted a core activity in the research lesson. After the research lesson, the teachers talked about actual students' behaviors concerning the revisions and evaluated their significance.
Thinking with doubt (doubt): Instead of taking something for granted, looking at it with doubt/suspicion is important. This ability came to be a point of focus in the sessions on the development of the lesson plan. The teachers considered whether such thinking was observed in students' behaviors in the research lesson. In further analysis, in order to develop their framework for studying the students, they searched the literature for definitions and ideas on the significance of critical thinking.
Change in students (change): The nature of changes in students as a result of classroom teaching is of great concern. In the research lesson, characterization of changes in students in terms of critical thinking was sought. After the research lesson, the teachers talked about the changes in students, and in particular, a group of four students and a specific student in the group. They investigated possible reasons for the changes by examining multiple sources of data from multiple angles.

Our analysis revealed that these four perspectives did not all develop at the same time. Different but related perspectives emerged in different situations and with different timing. Table 3 provides a summary of the earliest instances in which each of these perspectives was identified through key words in the resume prepared by the two teachers in the 20 sessions. In the discussions on developing a lesson plan, three perspectives appeared as follows: The perspective on "language" appeared in session 2; the perspective on "revision" likewise appeared early; the perspectives on "doubt" appeared a short while later. The perspective on "change" appeared only after the research lesson, when the teachers discussed students' thinking and learning.

In these discussions, earlier perspectives were referenced to aid in understanding students' behaviors. However, the perspective on "doubt" became predominant in the discussions when the teachers were engaged in the activity of analyzing students' learning during the research lesson. Furthermore, it came to be connected to the perspective of "change." The perspective of "language" then came to be referenced as another perspective relevant to that of "change." In the teachers' final report, the perspectives of "doubt" and "language" were used as core concepts to describe the students' learning through the research lesson.

Table 3. Emergence of the four perspectives in the activities of the two teachers

Month	Session no.	Activity	First appearance of key words in the resume by the teachers	Emergence of perspective
October	1			
November	2		-Using mathematical expression	*Using the language of mathematics
	3		-Recreation	
	4	Developing the lesson plan	-Making and improving instruction manual	*Revising the initial method to create a better one
	5			
December	6		-Evaluate and improve; eyes of doubt/ suspicion	
	7			
	8		-Critical thinking; self- and peer-evaluation	*Thinking with doubt
	9	Conducting the research lesson		
	10	Reflecting on the research lesson through interviews with students		
	11			
	12		-Student's noticing; deepening understanding ("fall straight down")	*Changes in students
January	13			
	14	Analyzing students' learning during the research lesson		
February	15			
	16		-Definition and categories of critical thinking	
	17			
	18		-G4 students' shift in critical thinking	
March	19	Writing a final report		
	20	Reflection		

There appear to be at least three incentives that stimulated the emergence and development of perspectives. First, the activity of developing the lesson plan provided a rich opportunity because many of the perspectives emerged during the discussion on the goal and development of the research lesson. The second incentive was the opportunity to analyze students' actual behaviors and learning in the research lesson based on the data. As described in the next section, sessions 10–12 afforded a fresh and valuable opportunity for teachers to draw a contrast between their prior expectations and the reality of the classroom and students. The third incentive was embodied in the activity of creating and assigning codes. In sessions 13–18, the researchers and the two teachers discussed the thinking and learning taking place in a group of four students during the research lesson. The most in-depth discussion centered on the codes should be assigned to the interactions among the students. In session 16, the two teachers first proposed codes that predominantly used terms taken from previous research by Japanese researchers on critical thinking. Since the teachers were not confident of their understanding of the terms or of the validity of assigning those codes, it decided that a bottom-up approach should be taken. Rather than borrowing names for the codes, we looked into the data closely and attempted to develop codes that accurately represented the content of students' interaction. This work led to rich discussion among all of the members of our group.

Processes of contextualizing during discussion of students' behaviors in the research lesson

In this section, we examine sessions 10 and 12. In these sessions, Mr. T, Mr. M, and one of the researchers (K) reflected on the research lesson and discussed the behaviors of students in different groups. They discussed one of the students' groups (G4) in particular, because the researcher had very closely observed and recorded the behaviors of this group, and in addition, the group had exhibited notable progress during the lessons. Our focus in this section is on the manner in which the teachers and the researcher participated in the discussion, and especially on how they used or proposed the four perspectives to maintain or extend the discussion. We also illustrate the teachers' processes of contextualizing the situation by providing excerpts.

Using and proposing perspectives by discussion participants

A qualitative analysis showed that both the teachers and the researcher used and proposed perspectives to maintain or to extend the discussion. Some of their usages were common to both the teacher and the researcher, while some were unique to each of them. Table 4 shows their behaviors regarding the use of perspectives to participate in the discussion.

Table 4. Behaviors of discussion participants regarding perspectives in sessions 10 and 12

Common code	Code for the researcher only	Code for the teachers only
- Interpretation of situation from perspective - Asking questions about the reasons for students' behaviors - Proposal of issues or explanation from interview data	- Proposal of perspective - Providing terms and information relevant to perspective - Proposal of method of interviewing	- Interpretation of other episodes from perspective - Providing information on students - Providing general information on the teaching and textbook - Explanation of the intention of specific teaching acts in the lesson - Reflection on teaching

The common codes show that the three members of the group examined and interpreted specific situations from certain perspectives that had already emerged, probed the situation by asking questions, or proposed further issues from the new information obtained from student interviews. These opportunities encompassed coordination between the perspectives and the newly encountered situations, or in other words, the process of recontextualization.

Other codes show more unique discursive acts on the part of the researcher and the teachers. The researcher recalled or proposed the perspectives when talking about the behaviors of the students in the lesson, supplied terms in the field of research in mathematics education and evaluated the significance of a specific situation from a particular perspective. It was also the researcher who provided background information relevant to a perspective. There were occasions when the researcher went beyond the specific situation to talk about other concepts related to the perspective or to make reference to a larger theoretical context. These contributions can be perceived as the process of decontextualization on the part of the

researcher. Furthermore, the researcher proposed a method of interviewing in session 10 which involved listing specific questions and giving reasons for the questions. The researcher thus appeared to be demonstrating an inquirer model.

On the other hand, the teachers positively recognized and interpreted other situations from the proposed perspectives, which again contained the process of recontextualization. The teachers played the most active roles in providing information on the students, teaching, and textbook. Mr. T, who taught the research lesson, knew each student well. The teachers also provided general information on teaching and textbook use. Furthermore, when asked to do so by the researcher, Mr. T provided information on the intention of specific teaching acts in the research lesson.

Below, we provide excerpts to show how the process of contextualization was observed in the practice of the teachers. All the excerpts were concerned with the perspective "using the language of mathematics."

Recontextualizing students' behaviors from a specific perspective

The first subject of discussion on students' behaviors in session 10 was that the students tended to forget the goal of the lesson. Mr. T initially referred to this fact when he spoke about his teaching in the research lesson. Next, the researcher mentioned his observation of G4 students. He explained what G4 students were doing and gradually recontextualized the situation from the perspective of "using the language of mathematics."

K: Students in G4 did not care for the goal of the lesson, either. Then they were told by the teacher about that. G4 was rather conscious of an easier way [to recreate the figure] from the beginning. … They didn't use rotational transformation or symmetrical transformation at all. They didn't use them, they left them totally unnoticed. [What they were doing was] really like geometrical construction at the elementary school level. Then when the teacher told them [about the goal of using the language of transformation], they did it.

…

K: (K picked up on the situation in which G4 students came to recognize that they did not use the language of transformation.) Here, G4 students began to talk about where the transformation was [in their instruction manual]. They recognized that this was the part and then said, "Oh, here? Yeah, you are right. Rather than being told 'upper or lower,' it is much more trivial for other people to be told 'symmetrical transformation.'" They said it is trivial.

T: I see.

K: In short, here, they were still just at a level of feeling.

T: Feeling of how the triangles are arranged.

K: When there is some ambiguity, by using the language of mathematics, [the ambiguity comes to be clarified because] everybody knows the language of mathematics. G4 students seem to develop such an attitude. The language [of mathematics] does not have ambiguity, and has well-defined words. When saying 'symmetrical transformation,' there exists a pattern. …

T: I see.

In this discussion of the first situation, it was the researcher who engaged in the process of recontextualizing and added new meaning to a given situation; the teachers participated in this process by nodding and by repeating, "I see."

The process of recontextualizing was observed not only on the part of the researcher but also on the part of the teachers. One such occasion was when the teachers examined situations in other groups or involving other students from the perspective they were discussing. For example, after the researcher contextualized G4's processing, as above, Mr. M mentioned that G2 students had shown similar thinking in their use of the language of transformation of geometrical figures. In this situation, the teachers interpreted the students' behaviors from the perspective of "using the language of mathematics", as the following excerpt shows:

M: In the beginning stage, when I was looking at Lesson 1, well, G1, or G2, I wonder which group it was… but there seem to have been many groups that did not use [the words of] transformation.

T: With at least three [groups] in the beginning, I thought, "What? [they are not using the words of transformation]."

M: Yes. You were mumbling something like that. I remember that G1, G2, G4 students were not using [the words of transformation].

….

M: At first, [G2 students] wrote around here. Then they began to write this part. They began to write "transform the figure by the rotation of 60 degrees". Here is the place that they came to use rotational transformation. I feel that the students changed to this direction because of listening to Mr. T's reminder of transformation of figures.

K: I see.

M: I can see similar observations in G3. They started with the construction of a hexagon, but then they changed to the direction of rotating [a piece of triangle] to make [the hexagon] at the end.

After these interactions, Mr. T immediately suggested reasons for the students' responses based on his teaching experience, explaining thus: "Students are taught that way [of constructing right hexagon] from the textbook. Perhaps they have been taught the content recently, and had a strong impression."

Applying a perspective to external situations: a process of decontextualization

Contextualizing was further observed when the teachers referred to other lessons or other mathematics content in relation to their teaching experience. They *desituationalized* (van Oers, 1998) the perspective and applied it to external situations. One such occasion was observed when the researcher and the two teachers again discussed G4's behaviors from the perspective of "using the language of mathematics." The researcher talked about G4 students' emergent use of transformations of geometrical figures in terms of a communication tool. He suggested that the language of transformations of figures provided keywords enabling students in other groups to connect different parts of the construction process, even though the G4 students were able to make this connection within themselves. At this point, Mr. T entered the conversation.

M: Students are not accustomed to logic. I have similar experience in the teaching of proof. Students are not accustomed to outputting everything that they have already understood. In short, they must write everything that they understand. Students shortcut the statements when they know 'this is because of this,' or something like that. From the side of the reader, the logic jumps, there are leaps. I think such [jumps] surfaced here. [The jump by the teacher] being pointed out, they might become conscious that "we must write precisely how to draw its center and so on."

Mr. M also recalled his experience of teaching mathematical proof by applying the perspective beyond the situation. In response to this, the teachers talked in depth about what their students were thinking. It is probable that they were applying the perspective more specifically in order to make sense of it more fully.

Deepening understanding of the situation by layering multiple recontextualizations

In another instance, a situation was recontextualized through the use of multiple perspectives. The interaction below took place in session 12. The researcher and the two teachers discussed the behaviors of students on the basis of the student interview data.

M: (Looking at a transcript of a student interview) She said, "The advantage of symmetrical transformation is, upper and lower, …" From here it begins.

K: (Reading the transcript) She said, "For those who do not understand what we write simply, [we use] words we have already learned."

M: It seems that by imagining other people, and if she uses the expression "symmetrical transformation," they can develop common understanding.

K: I see.

M: She remembered that "upper and lower" could not convey her intention. In the activity of mutual evaluation, her friends evaluated her group's instruction manual by saying something like, "what does 'image of upper and lower vertex' mean? I don't know what the vertex is." So she understood that the words "upper and lower" are not effective to convey the meaning. And she thought that the word of symmetrical transformation would be better to convey meaning because all class members did learn the word.

K: I see. This is almost as if she was saying mathematics is a common language.

T: I think this part [of the transcript] is similar. … The student said, "the words that we were saying within our group were not heard by the people in other groups." Even if they thought that they could understand it sufficiently, …

K: I see. This is interesting.

T: So we asked the students [in the interview] in what way they had learned symmetrical transformation, because the student said, "Everyone learned it in the same way."

K: Oh, it's great.

M: This is exactly the power of evaluation. The activity of mutual evaluation among students brought to this kind of awareness.

K: Great.

T: Both of the students were saying that in Lesson 1, they believed their use of words was all right. However, it was overbearing confidence without any reason.

The transcripts of the interviews provided the teachers with opportunities to add meaning to the situation from the perspective of "using the language of mathematics." Moreover, Mr. M pointed out the influence that the students' activity of mutual evaluation in the lesson had had on their noticing of the usefulness of the language of mathematics as a communication tool (see the underlined utterance in the excerpt). In terms of contextualizing, Mr. M recontextualized the same situation from the different perspective of "changes in students." This indicates a beginning in understanding a situation through multiple layers of perspectives.

Shortly after these interactions, Mr. T said, "It seems to me that the students made sense of it. (He expressed it by "fall straight down.") Such understanding emerged not because they were told by someone. I really felt that." Mr. T seems to have recognized the educative power of organizing students' activities based on these processes of recontextualizing rather than on the teacher's instruction.

The teachers' reflections on participation in our MPD program

Session 20 was devoted to the two teachers' overall reflections on their participation in our program. Both of them listed the following three activities as being the most impressive: engagement in coursework for both undergraduate and graduate students, participation in lesson studies in local schools, and conducting practical research. The impact of practical research was especially significant. Mr. T said, "From the stage of making the lesson plan, we thought about research that will continue to develop after the lesson. Together we elaborated (he used a Japanese word, "tataku," which means "hit") the lesson plan again and again, which became a big asset both for me alone and for us."

Both of them stated that through their practical research, they had learned to deal with an issue by focusing on a target object. Mr. T said, "I knew that research extends forever. So, I felt it is important to decide on at least one target, and to aim at and attack it." Mr. M said, "I know my experience is just the very beginning, but I knew how to do research by deciding on a theme…. focusing on an object, conducting a lesson with specific intention, and making analysis. I could have an image of how to do such things." These statements suggest that the teachers had developed a stance of doing research in teaching practice.

The two teachers also reflected on the cycle of LS that they had engaged in. They stated that in the beginning, they had been constructing the lesson plan in a somewhat vague manner by trying to incorporate a process of evaluation/revision into students' activities. However, when they analyzed the students after the lesson, they recognized that the concept of students' critical thinking was of significant concern, and they became aware of the need to pay attention to the thinking of students in more detail. It was their first experience of analyzing students' thinking by means of the collected data. Mr. M said, "[After the lesson] our theme became really focused." Reflecting on the shift from pre-existing codes to more bottom-up codes, Mr. T said, "To me, I felt like that [the analysis] became our property." We found that the teachers experienced several shifts as they engaged in the LS cycle. Both teachers mentioned that all the different activities they had engaged in during the program had led to such shifts.

The teachers further discussed the implications of their experience of practical research for their teaching practice. Both teachers reflected on their failure to fully notice students' behaviors and thinking in

the lesson. Mr. T said, "I really felt that there is something in the students, although we may not know there is." In session 10, the teachers had already mentioned that students were thinking more than was expected. These utterances indicate certain changes with respect to the two teachers' view of students. Moreover, Mr. M said that the ideas they incorporated into the research lesson could be applied to lessons on other mathematical content. He especially pointed out the possibility of implementing these ideas for improvement in his teaching of mathematical proof.

DISCUSSION

In this paper, we examined mid-career teachers' learning in a practical research activity in an MPD program. The two teachers engaged in the creation of an MLF, or in other words, a model of teaching and learning in a mathematics lesson that would foster students' abilities to think, make decisions or express their thinking. We illustrated the emergence and development of the teachers' perspectives that constituted the MLF. Different perspectives emerged at different periods and during different activities and gradually evolved or came to relate to each other. The perspective of "language" seemed to be concerned with the goal in teaching the lesson, while the perspective of "revision" was more concerned with the development of the lesson. On the other hand, the perspective of "change" was concerned with the result or outcome of the lesson. Therefore, the various perspectives provided opportunities for the teachers to understand pedagogical situations from multiple angles and layers.

The perspective of "doubt" first emerged as a goal of teaching the research lesson, but later developed as a means of analyzing students' thinking and learning in the research lesson. "Doubt" also came to relate to the perspective of "change." It is noteworthy that this perspective has a connection to research, and in particular, to research on critical thinking in mathematics education. The teachers developed an interest in viewing students' thinking and learning from this perspective, and began to read research literature on critical thinking. As described earlier, they worked hard to develop codes to identify the features of critical thinking among a group of students in the research lesson, which seems to be a process similar to that described in Hennessy and Deaney (2009). For the teachers, deriving findings on student's critical thinking became a key construct in their final report. Moreover, after completing the program, they presented their work at conferences and teacher meetings (e.g., Takita, Mizuno, Kawakami, Makino, & Hino, 2019). In this we can observe the functioning of infrastructures for professional exchange and publication of teacher knowledge (Miyakawa & Winsløw, 2019). The development of this perspective indicates that the teachers are likely to make sense of the significance of research in their practical research activity.

Contextualizing was also highlighted, as it served as a lens to look closely at how creation of an MLF is promoted by the interplay between teachers' perspectives and different situations in teaching practices. Consistent with Lin et al. (2018), this led us to underscore the processes of recontextualizing and decontextualizing. Although our analysis is limited to a small portion of the session discussions, the results show that by recontextualizing, the teachers problematized the situation that they had been thinking obvious, and added new meanings to the situation. It was also found that not only the researcher but also the teachers were decontextualizing the situation, but the results further showed that the teachers' ways of decontextualizing were not the same as

those of the researcher. Through recontextualizing and decontextualizing, the teachers actively created connection and coordination between the perspectives and situations in their teaching practices. Notably, mid-career teachers have the ability to contextualize different situations encountered by them.

This stresses the analysis of student thinking and learning during the lesson as a critical opportunity for mid-career teachers to develop their professional knowledge. Goldsmith et al. (2014) posit "attention to student thinking" as one of six major areas of teacher learning. For the mid-career teachers, what they gained through the research activity was not just the idea of paying attention to student thinking but also an awareness of the need for analyzing student thinking and changes in thinking. This is consistent with the results of the survey we administered previously in which many participant teachers pointed out the significant impact of analysis of students on their views of students (Hino & Makino, 2012, 2014).

An analysis of teachers' learning in this study further revealed the critical role of university researchers in the discussions (see also Iwasaki & Miyakawa, 2015). The researchers proposed their own ideas and interests, which contributed greatly to the development of perspectives. It is also worth mentioning that the activity of lesson planning contributed significantly to the emergence of perspectives that would play a role in later activities. In the later activities, the teachers devoted themselves more to coordinating such perspectives to deepen understanding of students' behaviors. Furthermore, based on the reflections of the teachers in the last session, engaging in different activities outside the research activity, such as joining in school visits or participating in graduate seminars, seems to have affected a change in their views on the teaching and learning of mathematics.

Finally, we address the importance of the problems and interests the teachers themselves brought into the MPD program (see also Makino & Hino, 2018). The two teachers had idiosyncratic experiences, problems, and interests with respect to mathematics teaching. We found that the problems and interests they brought into our program continued to play a role while they engaged in the practical research. Mr. T's interest in students' interaction and mutual learning seems to support his challenge and elicited surprise when he was conducting and analyzing the research lesson. He stated that he had not previously thought of a lesson of collaborative learning on such a deep level. He also said that he did not know that his students could actually learn from each other spontaneously. On the other hand, Mr. M had a particular interest in geometrical proof and began to plan his research lesson around that content. Even when he joined Mr. T's lesson planning, his interest in proof continued. His personal notes showed that he kept connecting his observations of Mr. T's lesson to the teaching of proof. One of the excerpts above also suggests such a connection. Mr. M, while he did not conduct the lesson, also developed his own learning by being involved in the collaborative activity of lesson planning and data analysis.

FINAL REMARKS

This paper investigated the mid-career learning of two teachers as they engaged in the creation of an MLF through a practical research activity. Their learning is characterized by an increase and enrichment of their perspectives. The growth of the teachers' perspectives was made possible by their active involvement in the processes of contextualizing a situation by applying the perspectives. It is also notable that different

activities of practical research together with intense discussions with university researchers stimulated the processes of contextualizing and contributed to the development of perspectives. These findings provide a means of examining from the viewpoint of teachers the interplay between theory and teaching practice that plays an important role in their development of professional knowledge.

This paper introduced many issues that need to be explored further. First, since this paper deals with only two teachers, more case studies are needed to nurture the constructs of perspective and contextualizing. Second, the strategy of improving MPD programs by utilizing the findings of this paper deserves further exploration. In particular, the role of MTE-Rs in coordinating and selecting theory that meets the need of teachers is worth pursuing, as pointed out by Lin et al. (2018). Third, to understand teachers' professional knowledge, further inquiry is needed to investigate the relationship between perspective and the body of theoretical constructs that exists. We also need to elucidate how perspective relates to teachers' *informal theory* (Sekiguchi, 2014). Fourth, follow-up should be done to determine the extent to which the teachers' perspectives, developed during the MPD program, will have an effect when they go back to their daily teaching practices and will shape their growth over time (Clarke & Hollingsworth, 2002). This task will relate to the challenging issue of creating scalable and sustainable PD program (Sztajn et al., 2017).

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